2.12

Binomial Theorem

The binomial coefficients that appear in the expansion of $(a + b)^n$ are the values of ${}_nC_r$ for $r = 0, 1, 2, 3,\ldots n$

A classical notation for ${}_nC_r$ especially in the context of binomial coefficients is $\binom{n}{r}$.

Both notations are read 'n choose r".

Using ${}_nC_r$ to Expand a Binomial:

Example:

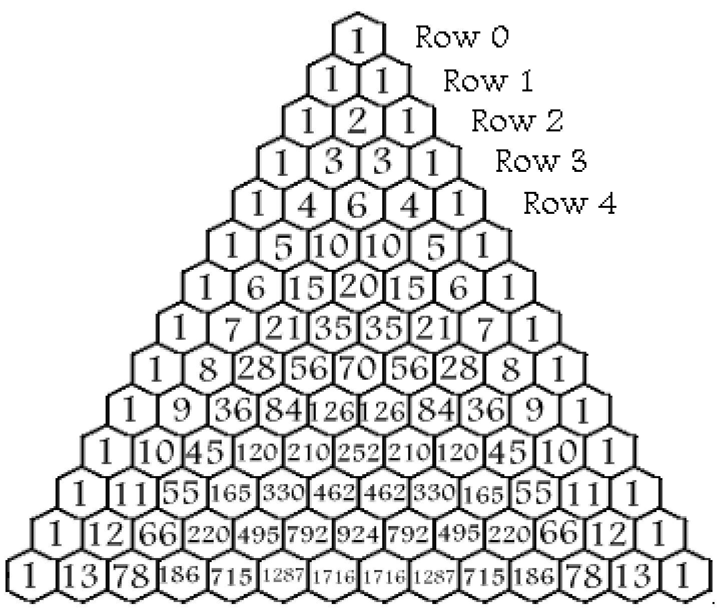
Expand $(a+b)^5$, using a calculator to compute the binomial coefficients.

Solution: Enter 5 ${}_nC_r$ {0, 1, 2, 3, 4, 5} into the calculator to find the binomial coefficients for n = 5. You should get the list {1, 5, 10, 10, 5, 1}.

Using these coefficients we get the expansion:

$$(a+b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$$

Pascal's triangle



The Binomial Theorem

For any positive integer n

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \ldots + \binom{n}{n}b^n$$

$$where \binom{n}{r} = {}_nC_r = \frac{n!}{r!(n-r)!}$$

Example: Expanding a Binomial

Expand $\left(2x-y^2\right)^4$ by hand using the Binomial Theorem.

Solution: a = 2x and b = $-y^2$.

Using: $\left(a+b\right)^4 = 1a^4+4a^3b+6a^2b^2+4ab^3+b^4$

$$\left(2x-y^2\right)^4 = \left(2x\right)^4 + 4(2x)^3\left(-y^2\right) + 6\left(2x\right)^2\left(-y^2\right)^2 + 4\left(2x\right)\left(-y^2\right)^3 + \left(-y^2\right)^4$$

$$= 16x^4 - 32x^3y^2 + 24x^2y^4 - 8xy^6 + y^8$$

Computing Binomial Coefficients:

Find the coefficient of x^{10} in the expansion of $(x + 2)^{15}$.

Solution: The only term in the expansion that we need to deal with is ${}_{15}C_{10}\, x^{10}2^5$.

$${}_{15}C_{10}\, x^{10}2^5 = 3003 \cdot 32 \cdot x^{10} = 96{,}096x^{10}.$$